Math 4FM3 Tutorial 2.

Ex 13.19.

$S_0 = 140$, $\sigma = 0.25$, $r = 0.04$, $\Delta t = \frac{3}{12} = 0.25$.

(a) $u = e^{\sigma\sqrt{\Delta t}} = e^{0.25\times\sqrt{0.25}} = 1.1331$, percentage up is 13.31%.

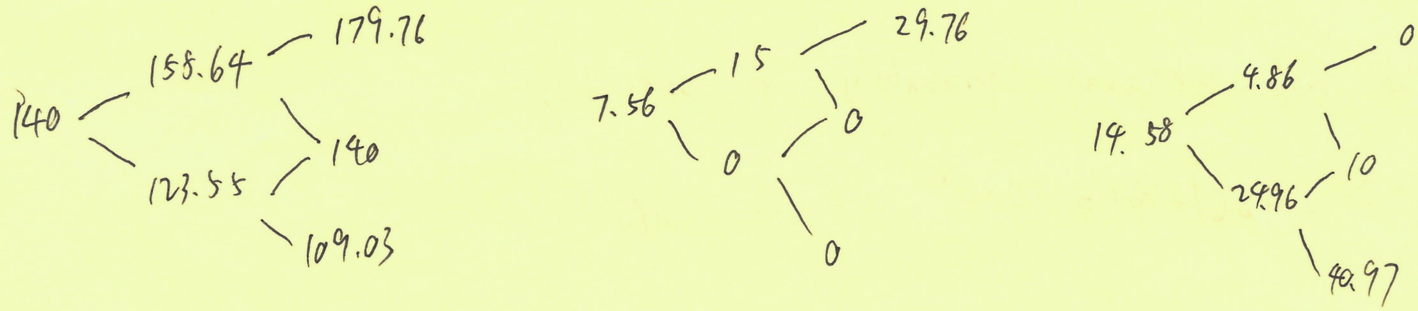
(b) $d = 1/u = 0.8825$. percentage down is 11.75%.

(c) $\tilde{p} = \dfrac{e^{r\Delta t} - d}{u-d} = \dfrac{e^{0.04\times 0.25} - 0.8825}{1.1331 - 0.8825} = 0.5089$

(d) $\tilde{q} = 1 - \tilde{p} = 0.4911$.

$K = 150$ for both European call and put.

Stock (Call & put). Call option Put option



Ex 13.20.

The delta for the first period $\Delta_1 = \dfrac{V_u - V_d}{S_u - S_d} = \dfrac{15}{158.64 - 123.55} = 0.4273$

$\Delta_2(H) = \dfrac{29.76}{179.76 - 140} = 0.7485$, $\Delta_2(T) = 0$.

Ex 13.21.

$S_0 = 50$, $S_u = 60$, $S_d = 42$, $K = 48$.

stock

$$\begin{tikzcd}[row sep=small] & 60 \\ 50 \arrow[ur, dash] \arrow[dr, dash] & \\ & 42 \end{tikzcd} \qquad \begin{tikzcd}[row sep=small] & 12 \\ V_0 \arrow[ur, dash] \arrow[dr, dash] & \\ & 0 \end{tikzcd}$$

Consider a portfolio consisting of Δ shares of stock and short an option.

then if no-arbitrage

$$42\Delta = 60\Delta - 12 \Rightarrow \Delta = \frac{2}{3}.$$

$42\Delta = 42 \times \frac{2}{3} = 28$. is portfolio value

Current value of portfolio is $(\frac{2}{3} \times 50 - f)e^{r/2} = 28 \Rightarrow f = 6.96$.

The risk-neutral probability $\tilde{p}$ satisfies

$$60\tilde{p} + 42(1 - \hat{p}) = 50 \times e^{r/2} \Rightarrow \hat{p} = 0.6161.$$